KATHMANDU UNIVERSITY
End Semester Examination [C]
December, 2024

Marks Scored:

Level : B.Tech.
Year : II

Course : AIMA 201
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 10

Registration No.:

Date : Dec. 12

SECTION "A"

[10Q. × 0.5 = 5 marks]

Fill in the blank space (s) by writing the most appropriate word(s) or symbol(s).

1. The degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt[3]{x+y}$ is _____.

2. The integrating factor of the differential equation $\frac{dy}{dx} + p(x)y = q(x)$ _____.

3. The differential equation $y' + p(x)y + q(x) = r(x)$ is homogeneous if _____.

4. The Laplace transform of $f(t) = t^5e^t$ is _____.

5. _____ is the inverse Laplace transform of $F(s) = \frac{1}{(s-1)}$.

6. The convolution of t and 1 is _____.

7. The root of the characteristic equation of a certain homogeneous differential equation are -1, 2, 2. Then the general solution of this differential equation is _____.

8. _____ is the differential equation whose solution is $y = cx^2$.

9. _____ is the general Euler-Cauchy equation where x and y are independent and dependent variables respectively.

10. The Wronskian of $y_1 = 1, y_2 = x$ and $y_3 = \frac{x^2}{2}$ is _____.

SECTION "B"

[10Q. × 0.5 = 5 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. Which of the following is not the integrating factor of the differential equation $xdy - ydx = 0$?
[$\frac{1}{x^2}$; xy; $\frac{1}{y^2}$; x]

12. When the method of undermined coefficient is used, the correct form of y_p for $y'' - 3y' + 2y = 5xe^x$ is _____.
[Ae^x; $(Ax + B)e^x$; Axe^x; $5xe^x$]

13. The partial differential equation $u_t = D(u_{xx} + u_{yy}), D \neq 0$ is known as _____.
[Heat equation; Wave equation; Laplace equation; Poisson's equation]

14. The system of differential equations $X'(t) = AX(t) + B$ is homogeneous if _____.
[$A = 0$; $B = 0$; $X(t) = 0$; $A = B$]

15. The inverse Laplace transform of $F(s) = \frac{1}{s^2+4}$ is _____.
[$\sin 2t$; $\cos 2t$; $\frac{1}{2}\sin 2t$; $\frac{1}{2}\cos 2t$]

16. If $y_1(t)$ and $y_2(t)$ are two solutions of second order linear differential equation, then Wronskin is given by $w(y_1, y_2) =$ _____.
[$y_1y_2' - y_1'y_2$; $y_1y_2 + y_1'y_2'$; $y_1y_2' + y_2y_1'$; $y_1'y_2' - y_1y_2$]

17. Which of the following is classified as parabolic partial differential equation? _____
[$u_{tt} - u_{xx} = 0$; $u_{tt} - c^2(u_{xx} + u_{yy}) = 0$; $u_t = D(u_{xx} + u_{yy})$; $u_{xx} + u_{yy} = 0$]

18. The Bernoulli's equation $\frac{dy}{dx} + p(x)y = q(x)y^n$ can be solved by first transforming it into a linear equation. Which of the following is the correct transformation? _____
[$v = y^n$; $v = y^{1-n}$; $v = y^{n-1}$; $v = \ln y$]

19. The system of linear equations $AX = 0$ where A is a square matrix always has _____.
[only one solution; trivial solution if $|A| = 0$; no solution; exactly one solution]

20. The Laplace transform of $\delta(t)$ is _____.
[0; 1; e^{-s}; $\frac{1}{s}$]

KATHMANDU UNIVERSITY
End Semester Examination [C]
December, 2024

Level : B.Tech. 12 - Dec. Course : AIMA 201
Year : II Semester : I
Time : 2 hrs. 30mins. F. M. : 40

SECTION "C"
[2 Q. × 6 = 12 marks]

1. Define linear differential equation. Determine whether the differential equation

$$dy + (2 - 3y)y\,dx = 0$$

is exact or not. If not, transform this to make it linear and solve. Also, find the particular solution for $y(0) = 1$. [1+1+3+1]

2. Define Laplace transform. Solve the differential equation: [1+5]

$$y'' + 3y' + 2y = f(t),$$

with initial conditions: $y(0) = 0,\ y'(0) = 0$and the function $f(t)$ defined as:

$$f(t) = \begin{cases} 4t, & 0 < t < 1 \\ 8, & if\ t > 1 \end{cases}$$

OR

State second shifting theorem. Also, solve the following differential equation using the convolution theorem:

$$y'' + 5y' + 6y = \delta(t-3), y(0) = 1, y'(0) = 0$$

where $\delta(t-3)$ is the Dirac delta function. [1+5]

SECTION "D"
[5Q. × 4 = 20 marks]

3. What do you mean by orthogonal trajectories? Find the orthogonal trajectories for the curve

$$y = cx^2$$

where c is arbitrary. [1+3]

4. Solve the differential equation $y'' + y = \sec x$ using variation of parameters method. [4]

5. Solve the following system of differential equations by substitution method: [4]

$$\frac{dy}{dt} = x + 5y$$

$$\frac{dx}{dt} = -x + 3y$$

P.T.O.

6. Find the solution $u(x, y)$ of the partial differential equation: [4]

$$u_x + 2u_y = 3xu$$

by separating the variables.

OR

Classify the following equations and solve by reducing to canonical form: [4]

$$u_{xx} - 6u_{xy} + 9u_{yy} = 0$$

7. A cup of coffee is initially at a temperature of 90°C, and the surrounding room temperature is 20°C. According to Newton's law of cooling, the rate of change of the temperature $T(t)$ of the coffee is proportional to the difference between the temperature of the coffee and the ambient temperature. The law is given by the differential equation:

$$\frac{dT}{dt} = -k(T - T_{env})$$

Where $T(t)$ is the temperature of the coffee at time t, $T_{env} = 20°C$ is the ambient temperature, k is the cooling constant, and $\frac{dT}{dt}$ is the rate of change of the temperature. Given that the initial temperature of the coffee is $T(0) = 90°C$, solve for T(t). [4]

SECTION "E"

[4Q. × 2 = 8 marks]

8. Solve $\frac{dy}{dx} = \frac{xy+y}{xy+x}$

9. Find the general solution of Euler-Cauchy equation $x^2y'' - 4xy' + 6y = 0$.

10. Show that inverse Laplace transform is linear.

11. Find the Laplace transform of $\sin at$.