KATHMANDU UNIVERSITY
End Semester Examination
September 2024

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| Level : B.Tech. | Course : AIMA 104 |
| Year : I | Semester : II |
| Time : 2 hrs. 30 mins. | F. M. : 55 |

22 sep 2024

SECTION "C"

[3Q. × 7 = 21 marks]

1. Define upper and lower triangular matrices. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, find $A^{-1}$ and show that $A^3 = A^{-1}$. [1+1+5]

**OR**

Write any system of linear equations with two variables having infinitely many solutions and no solution. Solve the following system of linear equations using Gauss Seidel method. [1+1+5]

$$5x_1 - x_2 + 2x_3 = 12$$
$$3x_1 + 8x_2 - 2x_3 = -25$$
$$x_1 + x_2 + 4x_3 = 6$$

2. State Gram – Schmidt process. Find the QR factorization of $A = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$ [1+6]

3. State Caley – Hamilton theorem and verify it for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ 2 & -4 & -4 \end{bmatrix}$ [1+6]

SECTION "D"

[6Q. × 4 = 24 marks]

4. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ by reducing into Echelon form. [4]

5. Use the properties of determinants to show: [4]

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b=c)^3$$

**P.T.O.**

6. Find the basis for null space of $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}$. [4]

**OR**

Find the basis for column space of $A = \begin{bmatrix} 1 & 4 & 8 & -1 \\ 2 & 4 & 8 & 0 \\ 3 & 2 & 4 & 1 \end{bmatrix}$. [4]

7. Let $\{\vec{v_1}, \vec{v_2}, \vec{v_3}\}$ is an orthogonal basis for $\mathbb{R}^3$ where $\vec{v}_1 = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix}$, then express vector $\vec{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$ as a linear combination of $\vec{v_1}$, $\vec{v_2}$ and $\vec{v_3}$. [4]

8. A transformation from the variables $X = (x_1, x_2, x_3)$ to $Y = (y_1, y_2, y_3)$ is given by $Y = AX$ and the other transformation from $Y = (y_1, y_2, y_3)$ to $Z = (z_1, z_2, z_3)$ by $Z = BY$ where $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -2 \\ -1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{bmatrix}$. Obtain the transformation from $X = (x_1, x_2, x_3)$ to $Z = (z_1, z_2, z_3)$. [4]

9. Let $T: V \rightarrow W$ be a linear transformation then prove that
   (i) $ker\, T$ is a subspace of $V$. [2]
   (ii) $im\, T$ is a subspace of W. [2]

SECTION "E"
[5Q. × 2 = 10 marks]

10. Find an acute angle between the vectors $\vec{u} = (1, 2, 2)$ and $\vec{v} = (5, 2\sqrt{2}, \sqrt{3})$.

11. Show that span of the vectors $\begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -6 \\ 0 \\ 2 \end{bmatrix}$ is one dimensional in $\mathbb{R}^4$.

12. If $\begin{bmatrix} -\frac{9}{2} \\ -4 \\ 1 \end{bmatrix}$ is an eigenvector of $A = \begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -4 \end{bmatrix}$, find the corresponding eigenvalue.

13. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a mapping defined by $T(x, y) = (2x - y,\ x)$ then test whether the mapping is linear or not?

14. Check whether the vectors $\vec{v_1} = \begin{bmatrix} 3/\sqrt{11} \\ 1/\sqrt{11} \\ 1/\sqrt{11} \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} -1/\sqrt{6} \\ 2/\sqrt{6} \\ 1/\sqrt{6} \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} -1/\sqrt{66} \\ -2/\sqrt{66} \\ 7/\sqrt{66} \end{bmatrix}$ are orthonormal or not?

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Registration No.:

Date :

## SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space (s) by writing the most appropriate word(s) or symbol(s).

1. A matrix $A$ is said to be an orthogonal matrix if ______________________.

2. A system of linear equations is said to be ______________________ and independent if the system of equations have exactly one solution.

3. If the inner product between the vectors vanishes then the angle between them is equal to ______________________.

4. If $A$ is a matrix and $\vec{u}$ is a vector such that $A\vec{u} = \vec{0}$ then $\vec{u}$ belongs to the ______________________ of $A$.

5. If $A = \begin{pmatrix} 3 & -7 \\ 1 & -2 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} -2 & a \\ b & 3 \end{pmatrix}$ then the value of $a + b$ is ______________________.

6. Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & -4 & 0 \\ 6 & 8 & 5 \end{bmatrix}$, then the sum of the roots of the characteristic equation of $A$ is ______________________.

7. If $\lambda$ is an eigenvalue of a matrix $A$, then ______________________ is an eigenvalue of $A^{-1}$.

8. Eigenvalues of the matrix $A = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$ are ______________________.

9. For a vector $\vec{u}$ in $\mathbb{R}^n$, consider a vector $\vec{y}$ in $\mathbb{R}^n$ such that $\vec{y} = \hat{y} + z$. If $z$ is the component of $\vec{y}$ orthogonal to $\vec{u}$ then $\hat{y}$ is called ______________________.

10. For an orthonormal set of vectors norm of each vector should be equal to ______________________.

SECTION "B"
[10Q × 1=10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The roots of $\begin{vmatrix} 1-x & 2 & 3 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = 0$ are________________.

[0 only; 1 only; 0 and 1; $-1, 0, 1$]

12. If $A$ is a square matrix such that $A^2 = A$ then the value of $(1-A)^3 + A$ is equal to ____________________.

[0; 1; $1-A$; $A-1$]

13. If $\vec{v_1}$ and $\vec{v_2}$ are the bases of the vectors $\vec{v_1} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ and $\vec{v_4} = \begin{bmatrix} 2 \\ 8 \\ 6 \end{bmatrix}$ of a vector space $W$, then what is the dimension of the vector space $W$?

[1; 2; 3; 4]

14. The set of vectors $\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$ are not __________________________.

[non-coplanar; linearly dependent; orthogonal; linearly independent]

15. Let $V$ and $W$ be vector spaces over $\mathbb{R}$ such that $T: V \to W$ be a map. Then T is linear transformation for a scalar $c$ if and only if______________________.

[$T(x+y) = T(x) + T(y),\ \forall\, x, y \in V$;

$T(x+y) = T(x) + T(y),\ \forall\, x, y \in W$;

$T(cx+cy) = cT(x) + cT(y),\ \forall\, x, y \in W$;

$T(cx+cy) = cT(x) + cT(y),\ \forall\, x, y \in V$]

16. Let $T: V \to W$ be a linear transformation then

(i) $ker\, T$ is a subspace of $W$ (ii) $im\, T$ is a subspace of $V$

Then which of the following is correct? _______________________.

[both (i) and (ii) are correct; both (i) and (ii) are incorrect; (i) is correct and (ii) is incorrect; (i) is incorrect and (ii) is correct]

17. If $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $T(1,0) = (2,3,1)$ and $T(1,1) = (3,0,2)$, then which of the following statement is correct? ________________.

[$T(x,y) = (x+y, 2x+y, 3x-3y)$; $T(x,y) = (2x+y, 3x-3y, x+y)$;
$T(x,y) = (2x-y, 3x+3y, x-y)$; $T(x,y) = (x-y, 2x-y, 3x+3y)$;

18. A square matrix $A$ and ________________ have the same eigenvalues.

[$A^T$; $A^{-1}$; $\frac{1}{A^T}$; $\frac{1}{A^{-1}}$]

19. Which of the following equations is true for the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$? ________________.

[$A^2 - 4A + 3I = 0$; $A^2 + 4A + 3I = 0$;
$A^2 - 4A - 3I = 0$; $A^2 + 4A - 3I = 0$]

20. The unit vector along $\vec{a} = (1, -, 2, 2, 0)$ is ________________.

[$\left(\frac{1}{9}, \frac{-2}{9}, \frac{2}{9}, 0,\right)$; $\left(\frac{1}{5}, \frac{-2}{5}, \frac{2}{5}, 0,\right)$; $\left(\frac{1}{9}, \frac{2}{9}, \frac{-2}{9}, 0,\right)$; $\left(\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}, 0,\right)$]