KATHMANDU UNIVERSITY
End Semester Examination
September 2024

Marks Scored:

Level : B.Tech.
Year : I

Course : AIMA 103
Semester : II

Exam Roll No. : Time: 30 mins. F. M. : 20

Registration No.: Date : 17 sep 2024

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space (s) by writing the most appropriate word(s) or symbol(s).

1. Suppose p and q are two statements. Then the statement $\sim p \Rightarrow \sim q$ is logically equivalent to the statement ____________________.

2. Let $f: G_1 \longrightarrow G_2$ be isomorphism between two graphs G_1 and G_2. If the sum of the degree of the vertices in G_1 is 30, then the number of edges in G_2 is ____________________.

3. If ρ is a permutation on a set A and k is the smallest positive integer such that $\rho^k = I_A$, then k is known as ____________________.

4. If a divides b and b divides a then $a =$ ____________

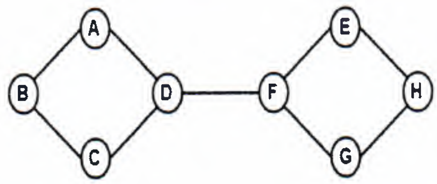
5. If we select 100 students from undergraduate students of KU randomly, then at least ____________ of them must have same birth month.

6. If $|A| = 3$ and $|B| = 4$, then the number of relations from a set A to B is ____________.

7. A Hamiltonian path in a connected graph includes every ____________ of the graph exactly once.

8. The maximum number of leaf nodes in a binary tree of height seven is ____________.

9. In a graph below, the cut vertices are ________________ is the bridge.



10. The number of odd vertices in the graph is always _______________.

SECTION "B"

[10Q × 1=10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The rule of inference "$((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p \Rightarrow r)$ is a tautology" is known as ______________________________.
[modus pones; modus tollens; law of detachment; law of syllogism]

12. If $A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$ is the adjacency matrix of a graph, then the maximum degree of a node in the graph is____________________________.
[2; 3; 4; 5]

13. Which of the following is not true about the permutation functions?
____
[The product of two even permutations is even;
The product of two odd permutations is even;
The product of two odd permutations is odd;
The product of or an even and an odd permutation is odd]

14. A connected graph G with 10 vertices is Hamiltonian graph if each vertex has degree greater than or equal to ______________________________.
[3; 5; 7; 10]

15. The matrix representation $M_R = [m_{ij}]$ of a reflexive relation R has the property ______________.
[$m_{ii} = 1$; $m_{ii} = -1$; $m_{ii} = 0$; $m_{ij} = 0$]

16. If $2, 2, 3, 3, 3, 3$ is a degree sequence of a graph G, then the number of edges in G are ______________________.
[4; 8; 16; 32]

17. The cardinality of R-relative set of $B = \{3, 5\}$ is ______________________ where R is the relation of divisibility on $A = \{1, 2, 3, 4, 5, 10, 20\}$.
[2; 3; 4; 0]

18. From a group of 7 men and 6 women, five persons are to be selected from a committee three men are there on the committee. Then the number of ways can it be done is______________________________.
[525; 735; 35; 840]

19. If f and g are real values functions defined by $f(x) = x^2$ and $g(x) = x^2 + 1$, then $(g\, o\, f)(-1) =$ ______________________________
[- 2; 0; 2; 3]

20. Traversing a binary tree by first visiting the left subtree, then the right subtree, and finally the root is called ____ traversal.
[pre-order; post-order; in-order; label order]

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SECTION "C"
[3Q. × 7 = 21 marks]

1. Define weak and strong form of mathematical induction. Prove by mathematical induction that if $A_1, A_2, \dots, A_n$ are $n \geq 2$ sets, then [1.5+1.5+4]

$$\overline{\left(\bigcup_{i=1}^{n} A_i\right)} = \bigcap_{i=1}^{n} \overline{A_i}$$

OR

Define logical equivalence. Prove that $\sim(\sim p \vee q) \wedge (p \vee q)$ and p are logically equivalent. Also, determine the validity of the following argument: [1+2+4]

If I drive to work, then I will arrive tired.

If I do not drive to work.

∴ I will not arrive tired.

2. Define equivalence relation. Show that $\mathcal{P} = \{\{1,3,5\},\{2,4\}\}$ is a partition of a set $A = \{1,2,3,4,5\}$. Find the equivalence relation determined by the partition $\mathcal{P}$ and draw the digraph. [1+2+2+2]

3. Define binary tree. Draw the expression tree from the postfix notation:

$$\mathbf{4\ 3\ 2 \div - 5 \times 4\ 2 \times 5 \times 3 \div \div}$$

and evaluate the value. Also, write this in prefix notation. [1+2+2+2]

SECTION "D"
[6Q. × 4 = 24 marks]

4. If R_1 and R_2 are two equivalence relations on a set A, then prove that $R_1 \cup R_2$ is a equivalence relation on A. [4]

OR

Let $A = \{1,2,3,4\}$ and $R = \{(1,2),(2,2),(3,4),(4,1)\}$ be a relation on A. Test the relation for reflexivity, symmetricity, anti-symmmetricity and transitivity. [1+1+1+1]

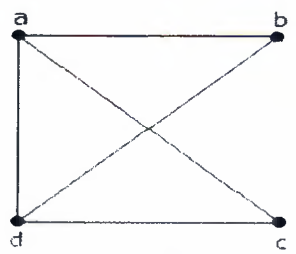
P.T.O.

5. Find GCD of 149553 and 177741. Also, write GCD as the linear combination of 149553 and 177741. [2+2]

6. Let $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ be a matrix representation of a relation R on a set $A = \{a, b, c, d, e\}$.

Find the digraphs from M_R and M_{R^2}. [1+3]

7. Find the number of spanning trees from the graph below. [4]



G

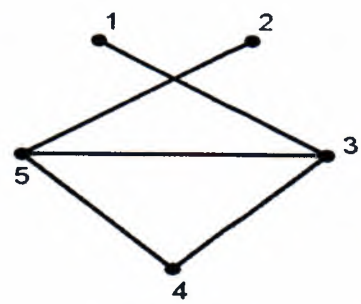
8. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of A. Then,

a. Write p as the product of the transpositions.

b. Find p^{-1}.

c. Find the period of p.

9. Show that the graph alongside is a complementary graph.



SECTION "E"
[5Q. × 2 = 10 marks]

10. Is it possible that the sum of the degree of the vertices of K_n to be 54? Justify your answer.

11. Write a note on eigenvector centrality of a node in a graph.

12. Draw two binary trees, each with 15 vertices: one with the maximum possible height and the other with the minimum possible height.

13. A group of 20 girls plucked a total of 200 oranges. Show that at least 24 oranges can be plucked by one of them.

14. Use one of the methods of proving to show that if n^2 is odd then n is odd.