KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2024

Marks Scored:

Level : B.Tech.
Year : I

Course : AIMA 102
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 7 April 2025

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space (s) by writing the most appropriate word(s) or symbol(s).

1. The Bowley's coefficient of skewness for the data set: 2, 4, 5, 7, 8, 9, 10, 12, 13, 15 is _____.

2. If the events A and B have equal probability and are independent with $P(A \cap B) = p(> 0)$, then $P(A) =$ _____.

3. _____ is a variable whose value is determined by the outcome of a random experiment.

4. In binomial distribution, if mean = 4 and variance = $\frac{4}{3}$, then $P(X = 0) =$ _____.

5. The normal distribution is also known as _____.

6. The joint probability mass function of two discrete random variables (X, Y) is given by $p(x, y) = k(x + y)$, (where $x = 0, 1, 2;\ y = 1, 2, 3$), then the value of $k =$ _____.

7. Let a joint probability mass function of X and Y be defined as $P(X = x, Y = y) = \frac{x+y}{30},\ x = 1,2;\ y = 1,2,3$. Then $P(X = 2, Y = 1) =$ _____.

8. The area of the confidence interval is _____ for α level of significance.

9. The probability value or tail area under the curve of the test statistic used in the hypothesis is called _____.

10. The least-squares regression line always passes through _____.

SECTION "B"
[10Q × 1=10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. _____ is most affected by outliers.
[Mean; Median; Mode; Range]

12. The data is positively skewed if _____.
[mean < mode < median; mode < median < mean;
median < mode < mean; mean < median < mode]

13. According to a survey of households, the probability that the residents own 2 cars if annual household income is over $25,000 is 80%. Of the households surveyed, 60% had incomes over $25,000 and 70% had 2 cars. Then the probability that the residents of a household have income over $25,000 is _____.
[0.2; 0.12; 0.785; 0.875]

14. If $X \sim B(n,p)$, then _____ is the standard deviation of X.
[$\sqrt{n^2p(1-p)}$; $\sqrt{np}$; $\sqrt{np(1-p)}$; $\sqrt{n(1-p)}$]

15. In standard normal distribution $N(0,1)$, the area that lies between -1.10 and -0.36 is _____.
[-0.2237; 0.2239; 0.2237; 0.4951]

16. Suppose that $p(x,y)$, the joint pmf of X and Y, is given by $P(0,0) = 0.4, p(0,1) = 0.2$, $p(1,0) = 0.1, p(1,1) = 0.3$. Then $P(X = 0 \mid Y = 1) =$ _____.
[4/5; 3/5; 2/5; 1/5]

17. If X and Y are independent, then their joint probability function must satisfy _____.
[$P(X|Y) = P(Y|X)$; $P(X,Y) = P(X)P(Y)$;
$P(X,Y) = P(X) + P(Y)$; $P(X|Y) = P(X)P(Y)$]

18. Suppose the heights of adult males in a population are normally distributed with a mean $\mu = 175$ cm and a standard deviation $\sigma = 10$ cm. Then _____ of the randomly selected male being between 165 cm and 185 cm.
[34.13%; 65.26%; 68.26%; 95.44%]

19. In testing of hypothesis _____ is known as *Type II Error*.
[Accept H_0 when it is true; Reject H_0 when it is true;
Accept H_0 when it false; Reject H_0 when it is false]

20. Correlation coefficient r is invariant under the changes of _____.
[origin and scale; origin and values; values and scale; b_{xy} and b_{yx}]

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7 April 2025

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F. M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. The following data represents the lifetimes (in hours) of a sample of 40 transistors.

112	121	126	108	141	104	136	134	121	118
143	116	108	122	127	140	113	117	126	130
134	120	131	133	118	125	151	147	137	140
132	119	110	124	132	152	135	130	136	128

a. Determine the sample mean, median and mode. [3]
b. Compute the sample variance. [2]
c. Plot a cumulative frequency plot of the data. [2]

2. Let X be a continuous random variable with probability density function given by

$$f(x) = \begin{cases} kx, & 0 \le x < 1; \\ k, & 1 \le x < 2; \\ -kx + 3x, & 2 \le x < 3; \\ 0, & \text{otherwise.} \end{cases}$$

a. Determine the constant k. [2]
b. Determine the cumulative distribution function (cdf) $F(x)$. [3]
c. What is the probability of the random variable less than 1.5? [2]

3. The following was collected from 10 students from last semester.

Assignment Grade (%)	85	35	60	79	100	90	41	65	95	71
Final Exam Grade (Out of 200)	136	40	88	125	165	145	52	98	155	111

a. Plot the data. [2]
b. Find the regression equation to predict the exam grade. [4]
c. Predict the exam grade of a student doing 50% homework. [1]

SECTION "D"
[6Q. × 4 = 24 marks]

4. State Baye's theorem. In a college 40% of students are girls. It is known that 15% of girls and 65% of boys are over 6 ft. tall. A tall student is just observed at distant horizon, what is the probability that it is a girl? [1+3]

5. Define expectation and variance of a random variable X. Suppose X is a random variable such that $E(X) = 3$ and $Var(X) = 5$. Let $H(X) = 2X - 7$, then find [1+3]

(a) $E[H(X)]$ and (b) $Var[H(X)]$.

6. In each of 25 races, the democrats have a 60% chance of winning.
   a. Find mean (μ) and standard deviation (σ) for $X \sim B(25, 0.6)$. [1]
   b. Using the normal approximation $N(\mu, \sigma^2)$ to the binomial (i.e. using the information from (a)), calculate the probability that democrats will win 19 or more races. [3]

7. Define multinomial distribution. If we roll a fair 6-sided die 10 times, what is the probability of getting 3 ones, 2 twos, 2 threes, 1 four, 1 five, and 1 six? [1+3]

8. Let X and Y (pressures for right and left tires) have the joint density function

$$f(x,y) = \begin{cases} \frac{1}{3000}(x+y), & 20 \le x, y \le 30; \\ 0, & \text{otherwise.} \end{cases}$$

   a. Determine the conditional density function $f_{Y|X}(y|x)$. [2]
   b. If the pressure in the right tire is found to be 12 psi, find the probability that the left tire has a pressure of at least 15 psi. [2]

9. An advertisement for a new toothpaste claims that it reduces cavities of children in their cavity-prone years. Cavities per year for this age group are normal with mean 3 and standard deviation 1. A study of 2,500 children who used this toothpaste found an average of 2.95 cavities per child. Assume that the standard deviation of the number of cavities of a child using this new toothpaste remains equal to 1. Are these data strong enough, at the 5 percent level of significance, to establish the claim of the toothpaste advertisement? [4]

SECTION "E"

[5Q. × 2 = 10 marks]

10. A problem in *Probability and Statistics* is given to three students A, B and C whose chance of solving it are $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{3}$, respectively. What is the probability that the problem will be solved? [2]

11. A basketball player makes 80% of his free throws. If he takes 5 shots, what is the probability they make exactly 3 shots? [2]

12. Show that the sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is the unbiased estimator of the population variance σ^2. [2]

13. Let $E[Y|X] = 5X - 1$ and $E[X] = 2$, find $E[Y]$ by using law of iterated expectation, i.e. $E[Y] = E\big[E[Y|X]\big]$. [2]

14. Calculate the coefficient of correlation if
$\sum X = 75, \sum Y = 80, \sum X^2 = 130, \sum Y^2 = 140, \sum XY = 120, n = 50.$ [2]