

KATHMANDU UNIVERSITY
End Semester Examination
March/April 2024

Marks Scored:

Level : B.Tech.

Course : AIMA 101

Year : I

Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date

22 April 2024

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space (s) by writing the most appropriate word(s) or symbol(s).

1. The domain and range of the sigmoid function $\frac{1}{1+e^{-x}}$ is _____
2. The function whose value at any number x is the greatest integer smaller than or equal to x is called _____
3. The value of $\lim_{x \rightarrow 2^-} \frac{|x-2|}{x-2}$ is _____
4. The vertical asymptote of the function $y = \frac{x}{x-2}$ is _____
5. The $\frac{dy}{dx}$ for $y = \tan^{-1} \sqrt{x}$ is _____
6. The graph of the function $f(x) = x^3 - 3x$ has horizontal tangents at the points where $x =$ _____
7. The average value of $f(x) = x$ on $[0,2]$ is _____.
8. If $y = \int_0^{x^2} \sin t \, dt$ then $\frac{dy}{dx} =$ _____
9. Domain of the function $z = \sqrt{x^2 + y^2 - 4}$ is _____
10. The value of $\int_1^2 \int_2^3 dx \, dy =$ _____

SECTION "B"
[10Q × 1=10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The function $f(x) = 1/x^2$ has a _____ discontinuity.
[removable; jump; infinite; oscillating]
12. The value of the limit $\lim_{x \rightarrow 0^+} [x]$ _____
[0; 1; -1; does not exist]
13. The largest interval where $f(x) = \sqrt{4 - x^2}$ is continuous is _____
[[-2,2]; [-2,2); (-2,2); (-2,2)]
14. Which of the following function is not smooth at $x = 1$ _____
[$y = x/(x - 2)$; $y = \frac{|x-1|}{x-1}$; $y = x - 1$; $y = x^2 - 1$]
15. The derivative of $y = x^x$ is _____
[$x^x \ln x$; $x^x(\ln x + 1)$; $x \ln x + x$; none]
16. $f'' = 0$ at a point implies f has a _____ at that point.
[point of inflection; critical point; no point of inflection; non of above]
17. The most suitable test for the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n}{(2n+1)!}$ is _____
[ratio; integral; root; nth term]
18. The nth term test for the series $\sum_{n=1}^{\infty} 1/n^2$ gives _____
[convergence; divergence; inconclusiveness; inapplicableness]
19. If f_x and f_y are the partial derivatives of $f(x, y)$ where $f(x, y) = c$ with respect to x and y respectively then $\frac{dy}{dx}$ is calculated by _____
[$-\frac{f_x}{f_y}$; $\frac{f_x}{f_y}$; $-\frac{f_y}{f_x}$; $\frac{f_y}{f_x}$]
20. The directional derivative of $f(x, y) = xy$ at $(0,0)$ along $\vec{u} = (1,1)$ _____
[0; 1; 2; 3]

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02-April-2025

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Draw diagrams where necessary.

SECTION "C"

[3Q. × 7 = 21 marks]

1. Define a point of inflection with an example. Sketch the curve of a function given (any one) writing all its necessary characteristics for sketching. [2+5]
a. $y = \frac{8}{x^2+4}$ b. $y = \frac{x^2+4}{2x}$
2. Find the length of the curve $y = x^2/2$, $0 \leq x \leq 2$. Find the surface area of the revolution of the solid formed by revolving the region $y = x^3/9$, $0 \leq x \leq 3$ around x -axis. [2+5].
3. Can a saddle point occur in a function of a single variable? Give reason. Find the absolute maximum and absolute minimum of the function $f(x, y) = 2 + 2x + 4y - x^2 - y^2$ on the triangular region in the first quadrant bounded by $x = 0$, $y = 0$, and $y = 9 - x$. [1+6]

OR

Find the gradient of the function $f(x, y) = \ln(x^2 + y^2)$ at (1,1). Find the extreme values and saddle points of $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$. [2+5]

SECTION "D"

[6Q. × 4 = 24 marks]

4. The mechanics at Lincoln Automotive are reboring a 6 inch deep cylinder to fit a new piston. The machine increases the cylinder's radius one-thousandth of an inch (1/1000) every 3 minutes. How rapidly is the cylinder volume increasing when the bore (diameter) is 3.800 inch? [Volume of a cylinder is $\pi r^2 h$]

5. Evaluate

$$\int \frac{2x^3 - 2x^2 + 1}{x^2 - x} dx$$

P.T.O.

6. Find the Taylor series of the function $f(x) = \sin x$ at $x = a$, and use this to find the Maclaurin series of $y = x^2 \sin x$. [3+1]

OR

Can you apply the Ratio test to the series $\sum_{n=1}^{\infty} a_n$ where $a_n = \begin{cases} \frac{n}{2^n} & n \text{ odd} \\ \frac{1}{2^n} & n \text{ even} \end{cases}$. If yes determine its convergence using the Ratio test, else use a suitable test to determine its convergence. [1+3]

7. Use chain rule to find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ for $z = \tan^{-1}\left(\frac{x}{y}\right)$, $x = u \cos v$, $y = u \sin v$.
8. Find the volume of the prism whose base is the triangle in the $x - y$ plane bounded by the lines $y = 0$, $x = 1$, $y = x$ and whose top is the plane $x + y + z = 3$.
9. Evaluate: $\int_0^1 \int_y^1 x^2 e^{xy} dy dx$. Is the change of order of integration necessary to integrate this integral?

SECTION "E"

[5Q. \times 2 = 10 marks]

10. Find a continuous extension of $(x) = \frac{x^2 + x - 6}{x^2 - 4}$.
11. Find the linearization of $f(x) = (1 + x)^k$ at $x = 0$ where k is a constant.
12. The length of a cube changes from 2.00m to 2.02m. Estimate the resulting change in its volume.
13. Show that $f(x)$ is big-oh of $g(x)$ where $f(x) = x^2 + x$ and $g(x) = x^2$.
14. Evaluate the Improper Integral $\int_{-1}^1 \frac{dx}{x^{2/3}}$.